Evaluation of Cumulative Damage Models for Fatigue Crack Growth in an Aircraft Engine Alloy

T. Nicholas*
Air Force Wright Aeronautical Laboratories, Wright-Patterson Air Force Base, Ohio
G. K. Haritos†
Air Force Institute of Technology, Wright-Patterson Air Force Base, Ohio
and
J. R. Christoff‡
Air Force Armament Laboratory, Eglin Air Force Base, Florida

Two models for evaluating crack growth in aircraft engine alloys under typical mission spectra were evaluated. Each model had the capability to determine the effects of frequency, stress ratio, temperature, and hold time on the crack growth rate. Data on an advanced alloy (AF115) were used to evaluate the hyperbolic sine (SINH) model and modified sigmoidal equation (MSE) model. Both models were found to have adequate capability and flexibility in modeling crack growth behavior over a wide range of conditions. The SINH model has been much more fully developed than the MSE model and is easier to apply to new materials.

Introduction

THE ability to predict fatigue crack growth rates in structural alloys has become an integral part of the design and life management procedures for U. S. Air Force gas turbine engines. Increased performance requirements resulting in higher operating temperature and stresses over the last two decades have resulted in drastic changes in the design criteria applied to critical engine components. In the 1960s, most critical structural components (e.g., turbine disks, spacers, etc.) were limited by creep and stress rupture properties. Less than 1% of all rotating components were life limited by low-cycle fatigue. However, today's designs find well over 75% of these components limited in life by low-cycle fatigue.

The present life management system for critical structural components whose life is limited by low-cycle fatigue is to retire these components from service at the end of their design life. The design life is determined by the time required to initiate a crack, therefore completely ignoring subcritical crack growth. Moreover, the design life is based on statistically safe material properties, using criteria such as 1 in 1000 components initiating a detectable crack. Statistically, 999 out of 1000 components will not have initiated a detectable crack when they are retired from service. Although this is clearly a conservative and safe design procedure, the replacement costs for these components are becoming prohibitive. Furthermore, analysis has shown that the average useful life of the components is often greater than 10 times their design life. Finally, the accurate determination of the tail end of the statistical distribution curve of component lives is very difficult, especially in cases where low-cycle fatigue life is heavily influenced by the presence of defects in the material. For these reasons, the Air Force is implementing a retirement-for-cause (RFC) life management philosophy on some of its existing engines.

Received Sept. 5, 1984; revision received Dec. 24, 1984. This paper is declared a work of the U.S. Government and therefore is in the public domain.

*Materials Research Engineer, AFWAL/MLLN.

†Associate Professor, Department of Aeronautics and Astronautics; Major, U. S. Air Force.

‡Aerospace Engineer, AFATL/DLCM; Lieutenant, U. S. Air Force.

The RFC approach calls for the retirement of components from service only after a crack is detected during periodic maintenance inspections. If no cracks are found, the component is returned to service for another inspection interval. This procedure requires accurate and reliable nondestructive inspection techniques to ensure detection of all cracks present. It also requires accurate modeling of crack growth to predict the time it takes for a crack to grow from the inspection limit size to failure. The frequency of the periodic inspections is established according to this prediction and it is generally required to be one-half or one-third of the predicted time-to-failure. Thus, safe operation is ensured by allowing one or two additional inspections to be performed during the predicted time-to-failure period.

In addition to adopting this RFC maintenance procedure, the Air Force has implemented the engine structural integrity program (ENSIP), which also requires a capability for predicting fatigue crack growth rates in engine components. While RFC requires an analysis of existing engine structural components, ENSIP addresses the design of new engines. It imposes a damage-tolerant design approach for all critical structural components in an engine. In this design process, a small initial fatigue crack is assumed to exist in all production parts. It is required that this initial flaw will not grow to a catastrophic size during the lifetime of the engine. Therefore, both RFC and ENSIP require accurate predictive capability for fatigue crack growth in engine materials under typical operating conditions. This paper presents an assessment of this capability for two state-of-the-art models that have been developed for crack growth rate predictions: one developed by General Electric Co. based on a modified sigmoidal equation and one developed by Pratt & Whitney Aircraft involving a hyperbolic sine equation. The models are discussed and demonstrated in a simple application to an advanced engine alloy and an assessment is made of their predictive capabilities.

Crack Growth Models

A crack growth model must have the capability to predict fatigue crack growth rates over a wide range of operational parameters. In addition to accurately modeling large amounts of experimental data, the ability to interpolate between experimental conditions is necessary. The approach common to

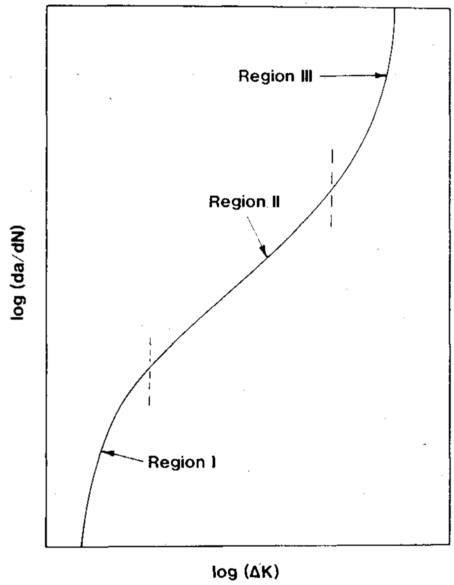

Fig. 1 Schematic of typical sigmoidal shape of crack growth curve.

both models assessed in this paper is to start with an equation that can represent the general sigmoidal shape of a typical fatigue crack growth rate curve, as depicted schematically in Fig. 1: region I represents the near-threshold or very low crack growth rate regime, region II the Paris law regime that is linear over some range for most materials, and region III the very high growth rate regime. The constants in the equation have to be determined as functions of the test variables. All of the work to date in modeling the fatigue crack growth rate in turbine engine materials has used the linear elastic fracture mechanics (LEFM) stress intensity factor, K, or the stress intensity factor range, ΔK, as the correlating parameter.[1-3] Although time- and/or frequency-dependent material behavior is often observed at the highest operating temperatures, this behavior is due to a synergism between environmental degradation (oxidation) and local time-dependent creep.[4,5] Under these conditions, K has been found to be an adequate correlating parameter, even for crack growth under sustained load.[6]

The test variables of primary concern in modeling crack growth in engine alloys are temperature T, cyclic frequency f, stress ratio R (the ratio of minimum to maximum applied stress), and sustained load hold time t_H. It should be noted that these are significantly different from the variables needed to adequately model crack growth in airframe or other room temperature alloys. There, the primary parameters of interest are the stress ratios and overload ratios in spectrum loading, while temperature, frequency, and hold time are not considered. Also, due to the nature of the engine load spectrum, overload effects are generally minor and are neglected in most crack growth modeling.[1] This paper evaluates the capability of two models to predict the fatigue crack growth rate in engine materials when only the frequency and the hold time are varied.

The MSE Model

A modified sigmoidal equation (MSE) was developed by General Electric Co.[7] to represent data for alloy AF115 in the form

$$da/dN = \exp(B)(\Delta K/\Delta K^*)^P\,[\ell n(\Delta K/\Delta K^*)]^Q\,[\ell n(\Delta K_c/\Delta K)]^D \tag{1}$$

The plot of this equation has the general sigmoidal shape shown in Fig. 1. Here, da/dN is the crack growth rate per cycle and ΔK the stress intensity range. The term ΔK^* represents the lower asymptote of Fig. 1 or the threshold stress intensity range, while ΔK_c is the upper asymptote or critical value of ΔK. The remaining constants B, P, Q and D can be related, in general, to the four test variables T, f, R, and t_H. The two asymptotes can also depend on these variables.

Table 1 Influence of experimental parameters on the sigmoidal model coefficients

Coefficient	Temperature	Frequency	Hold time	Stress ratio
ΔK_c				X
ΔK^*	X			X
ΔK_i	X	X	X	X
dA/dN_i	X	X	X	X
dA/dN_i'	X	X	X	X
Q	X			X

Equation (1) was developed for and applied to the crack growth rate predictions of alloy AF115, a high-temperature nickel-base turbine disk superalloy. A large body of experimental data was generated over ranges of the four test variables. Changing one or more of the test variables will result in a number of changes of the general curve of Fig. 1: 1) the inflection point of the curve may translate both horizontally or vertically, 2) the slope at the inflection point may change, 3) the shape or curvature of the upper and lower portions may be altered, and 4) the upper and lower asymptotes may shift. The six parameters control these changes. Parameter B controls the vertical motion of the entire curve. Parameter P provides control of the slope at the inflection point. The vertical location of the inflection point is controlled by a combination of B, P, and ΔK^*. The horizontal location of the inflection point is governed by a complex combination of a number of the parameters. The asymptotes have already been defined. Because of the complexity of all these interactions, an alternate set of six parameters was introduced to facilitate relating the movements of the curve to the values of the test variables. This new set of parameters, which is made up of three of the original ones (ΔK_c, ΔK^*, and Q), and of three new ones, and the dependence of each on the test variable, is shown in Table 1. An X indicates a functional relation. It can be seen, for example, that ΔK_c depends only on R, while ΔK_i, da/dN_i, and da/dN_i' are each related to all four test variables. These functional relationships were established for AF115 and involved either linear or power law dependence on the variables or their logarithms. The MSE equation has not been applied to other materials, so it is not known if these forms of functional dependence would hold for other materials. Nonetheless, in order to evaluate the predictive capability of the MSE model, the relationships were established for AF115 and incorporated into a computer program that provides numerical values over the range of variables used in this investigation.

The SINH Model

A model based on the hyperbolic sine (SINH) was developed by Pratt & Whitney Aircraft[8-11] to interpolate the crack growth rate data for several materials over ranges of the four test variables T, f, R, and t_H. It is expressed in the form

$$\log(da/dN) = C_1 \sinh[C_2(\log\Delta K + C_3)] + C_4 \tag{2}$$

which provides the basic sigmoidal shape of Fig. 1. Through the use of the logarithms of da/dN and ΔK in Eq. (2), the ability to physically interpret movement of the curve of Fig. 1 is enhanced. Note that Fig. 1 represents a sigmoidal curve in logarithmic coordinates. In Eq. (2), C_1 is a shape factor normally set equal to 0.5 for many materials. The constant C_2 is also a shape factor, while C_3 and C_4 control the horizontal and vertical location of the inflection point, respectively. The four constants can be related, in general, to the four test

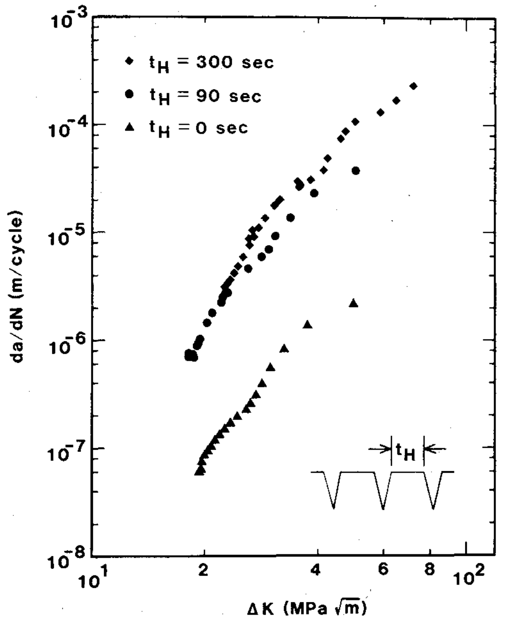

Fig. 2 Experimental data showing effect of hold time on crack growth in AF115 at 649°C, $R=0.1, f=0.25$ Hz (Ref. 7).

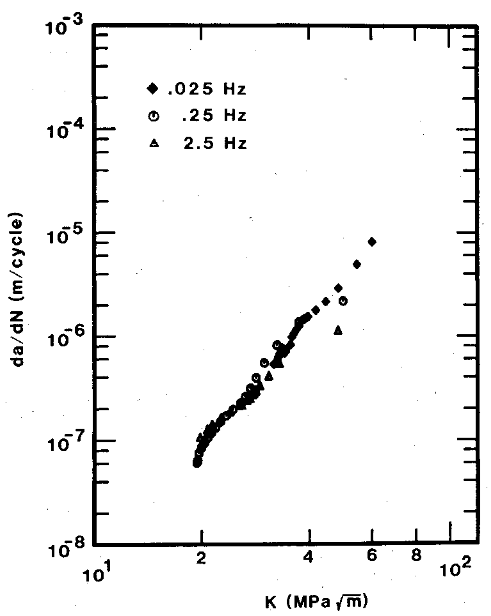

Fig. 3 Experimental data showing effect of frequency on crack growth in AF115 at 649°C, $R=0.1$ (Ref. 7).

variables. Through extensive application to a large body of data on several materials, the following general functional forms have been established for the SINH model:

$$C_1 = \text{const}(=0.5)$$

$$C_2 = a + b_1\log(1\text{-}R) + b_2\log(f) + b_3T + b_4\log(t_H)$$

$$C_4 = c + d_1\log(1\text{-}R) + d_2\log(f) + d_3T + d_4\log(t_H)$$

$$C_3 = e + fC_4 \qquad (3)$$

The 12 constants (a, b_1, b_2, $\cdots$, f) have to be determined for any given material. This procedure has been facilitated through the development of a sophisticated regression analysis that has been incorporated into a computer program with interactive capabilities. This allows one to assume values for some of the constants and to observe the fit to groups of experimental data points. Revised estimates of the constants can be introduced to achieve improved fits to experimental data.

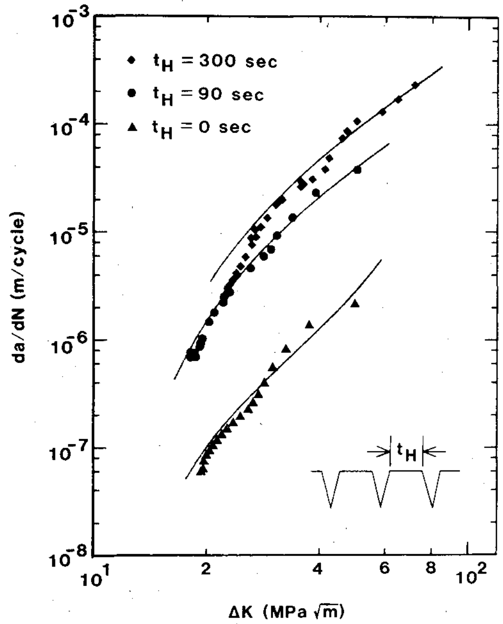

Fig. 4 Analytical predictions of MSE to hold-time data of Fig. 2.

Application of the Models to AF115

The capability of the two models to predict crack growth rates over a range of hold times and frequencies was evaluated using a limited set of data on AF115. Recall that the MSE model was developed from a large set of data on this material. Since there is no general procedure for application to other materials, the ability of this model to fit data generated for materials other than AF115 cannot be evaluted. Thus, the MSE model was not evaluated in a totally objective sense. Rather, its ability to fit a portion of the data used in its development and its interpolative capabilities were examined. On the other hand, the SINH model was evaluated according to its ability to fit this same limited data set (not used in its development) as well as its interpolative characteristics.

All AF115 data chosen for the model evaluation were at 649°C and $R=0.1$. The first set of data represents conditions of a cyclic frequency of 0.25 Hz with hold times at maximum load of 0, 90, and 300 s. These data, shown in Fig. 2, exhibit significant time-dependent behavior: the crack growth rate increases with increasing periods of hold time. The second set of data, representing frequencies of 0.025, 0.25 and 2.5 Hz with zero hold time, is shown in Fig. 3. Note that these data, collected at the same temperature of 649°C, do not exhibit time-dependent behavior and the crack growth rate does not vary significantly with frequency. These six data sets are not all independent. The data set at 0.25 Hz with zero hold time in Fig. 2 is, by definition, the set at 0.25 Hz in Fig. 3. Thus, five independent data sets were used in these evaluations.

The MSE model fit to the data of Fig. 2 is shown in Fig. 4. The values of the constants in the MSE equation are incorporated in the computer program for representing AF115. The constants which represent these equations are given in Table 2. The SINH model was fit to the three data sets of Fig. 2 using the regression analysis routine in the SINH computer program. The results are shown in Fig. 5 and the constants are tabulated in Table 3. The solid lines in this figure represent the fit obtained when all of the data points were included, while the dashed lines were obtained when the uppermost data point for $t_H=0$ was ignored. This is further discussed in the following section. It can be seen that both the MSE and SINH models provide a good representation of each of the data sets.

The fit of the two models to the frequency data of Fig. 3 was handled quite differently for illustrative purposes. The MSE model provided no variation with frequency and involves a single set of constants (see Table 2). The SINH model was fit to the three sets of data at different frequencies by treating them as three distinct and independent sets, even though they plot essentially on top of each other. The model had the capability to fit a different curve to each individual set of data, although the three curves are hardly distinguishable. Note that the constants for this fit for $f=0.25$ are different from those for the same frequency for a zero hold time, even though they both represent the same set of data. This point will be discussed further when the models are compared.

In addition to comparing the capability of each model to fit the two sets of experimental data, an evaluation was made of the interpolative aspects of each model within the two data sets. Choosing two values of K, each model was used to predict crack growth rates over the entire range of hold times of 0-300 s (0.25 Hz cycle) and for frequencies of 0.025-2.5 Hz with zero hold time. The results are summarized in Fig. 6 on a log-log scale. The cyclic data are plotted as crack growth rate vs cycle time (reciprocal of frequency). Several features are apparent from these curves. The two models predict essentially the same behavior over the entire range of hold times. However, the two models deviate slightly in their predictions of growth rate as a function of frequency. The MSE model predicts a linear variation of crack growth rate with cycle time or hold time, while the SINH model attempts to match the experimental data using a nonlinear functional relationship. In the particular case of the frequency data, the SINH model shows an apparent systematic nonlinear variation of growth rate with frequency. This apparent variability, however, is no larger than the inherent scatter in the experimental data. Thus, for these particular data sets, both models are equally valid for predicting growth rates as a function of hold time or frequency over the ranges tested.

Comparison of Models

Several common points and a number of differences became apparent in the course of this evaluation of the MSE and SINH models. Each started with an equation representing the general sigmoidal shape of a crack growth curve. The equation involves a number of parameters that are used to adjust the position, shape, and limits of each curve as a function of the four test variables. The MSE model was developed as an equation to determine da/dN, while the SINH determines $\log(da/dN)$. Similarly, ΔK and $\log(\Delta K)$ are independent variables in the MSE and SINH models, respectively. A study of the development and evolution of the functional relationships between the parameters and the test variables seems to indicate that the logarithmic formulation of the SINH model lends itself better to physical interpretation of translation and rotation of the curves on logarithmic plots than the linear formulation of the MSE model.

Evaluation of the functional form of the two models reveals that the MSE model provides for upper and lower asymptotes to represent the critical and threshold values of ΔK, respectively, while the SINH model makes no such provision. Providing

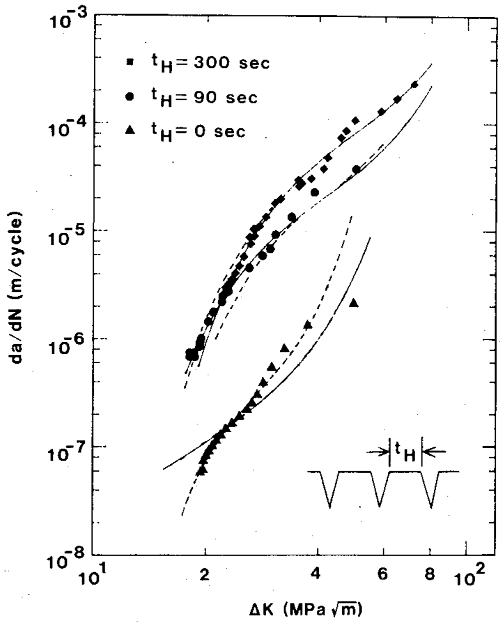

Fig. 5 Analytical predictions of SINH to hold-time data of Fig. 2 (solid lines show original fit, dashed lines revised fit).

Table 2 Constants for MSE model [Eq. (1)][a]

f,Hz	t_H, s	B	ΔK^*	ΔK_c	P	Q	D
0.25	0	−7.36	10	109.8	−2.27	3.00	−3.69
0.25	90	−7.99	10	109.8	0.15	3.00	−0.47
0.25	300	−7.61	10	109.8	0.46	3.00	−0.23
2.5	0			(Same as $f=0.25$)			
0.025	0			(Same as $f=0.25$)			

[a] da/dN given in inches/cycle for values of ΔK in ksi·in.$^{1/2}$.

Table 3 Constants for SINH model [Eq. (2)][a]

f,Hz	t_H, s	C_1	C_2	C_3	C_4
0.25	1[b]	0.5	4.787	−1.276	−5.315
0.25	90	0.5	5.146	−1.573	−3.100
0.25	300	0.5	5.242	−1.652	−2.508
0.25	1[b]	0.5	5.838	−1.513	−4.466
2.5	1[b]	0.5	4.691	−1.417	−4.924
0.025	1[b]	0.5	7.046	−1.609	−4.048
0.25[c]	1[b]	0.5	7.649	−1.397	−4.907
0.25[c]	90	0.5	5.548	−1.598	−3.014
0.25[c]	300	0.5	4.986	−1.652	−2.508

[a] da/dN given in inches/cycle for values of ΔK in ksi·in.$^{1/2}$.
[b] One second used as approximation for zero hold time for numerical purposes.
[c] Represents fit for dashed lines in Fig. 5.

for an asymptote, paticularly at the lower end, has the advantage that a true threshold can be modeled. However, this feature also has a disadvantage: when numerical procedures are used to obtain constants, the experimental data falling outside (below) the threshold value cannot be treated in regressing on crack growth rate. No work has been done with these models in the near-threshold regime of engine alloys, although the SINH model has been used successfully to represent near-threshold crack growth rate data at room temperature for an aluminum alloy.[12]

The major differences between the two models lie in their state of development and application. The MSE model has been developed for, and is based on, data on AF115. It has not been applied to other materials and procedures for determining the functional forms and their constants for application to other materials are not available. It is not clear whether the forms of the functional relations established in the MSE model for AF115 would be applicable to other materials. Judging by the success of the SINH model in applying the same functional forms [Eq. (3)] to a number of materials, we would recommend using the same types of relations as used on AF115 in further developments of the MSE model for application to other materials. The SINH model, on the other hand, has functional relationships that are well defined and whose constants can be obtained through the application of a computer program containing sophisticated regression analyses for fitting experimental data. Because of these differences, it is difficult to make a direct comparison of the capabilities of the two models.

The application of the SINH model can lead to different results by different users, depending on the manner in which the data are treated. In this evaluation, for example, the group of the three data sets for differing hold times was treated independently from the one containing the three data sets for differing frequencies, even though one set of data was common to both groups. This resulted in an apparent ambiguity for the case of a frequency of 0.25 Hz and a hold time of zero. The regression routine used to determine the constants as a function of hold time only involves $\log(t_H)$. Thus, a hold time of zero cannot be used. A value of 1 s was used for computational convenience. In determining the constants as a function of frequency, the frequency data set was used independently from the hold-time data. The computer program treated the data set common to both as independent entities in the hold-time and frequency regressions. The program could have been forced to yield the same results for $f = 0.25$ Hz in the frequency regression and $f = 0.25$, $t_H = 1$ in the hold-time regression. This was not done, however, in these exercises.

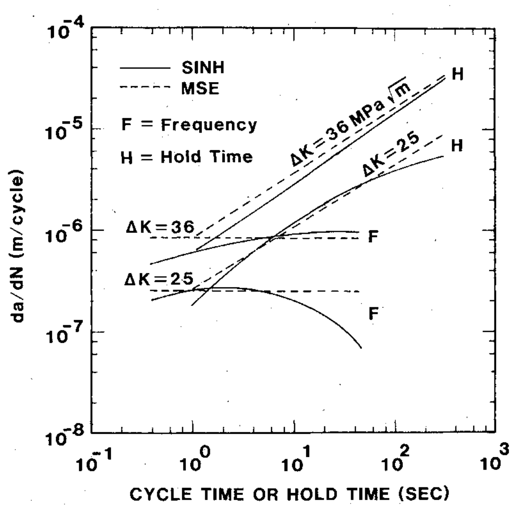

Fig. 6 Predictions of variation with frequency or hold time for MSE and SINH models.

Another approach that can change the results of the SINH modeling is the alteration of the data set representing the zero hold time. Figure 2 shows the data set used in the modeling. The uppermost point for $t_H = 0$ does not appear to be consistent with the remainder of the data. Eliminating this single data point and performing the regression analysis on the remainder of the data results in a revised fit as shown by the dashed lines in Fig. 5. Note the improvement in the fit of the zero hold-time data, while the fit to the upper two curves is not significantly affected.

Another procedure that could have been adopted would be to use all of the data sets for the three frequencies to represent the zero hold-time response in Fig. 2. Since there was no apparent frequency effect in this material at 649°C, this larger data set would bias the lower curve fit in Fig. 4 and probably provide a better overall fit. Finally, in any nonlinear regression routine, convergence of a numerical procedure is highly dependent on the inputs for initial guesses of the constants. In the application of the SINH model, the location of the inflection point is an initial guess. Different initial guesses could conceivably provide convergence to other final values of the constants. Familiarity with the program and the data as well as experience normally lead to improved predictive models.

Finally, the results of Fig. 6 demonstrate a feature of the models that may be important in future applications. The MSE model appears to provide a linear relationship between crack growth rate and cycle time on a logarithmic plot. The SINH model, while providing what appears to be added flexibility by allowing a nonlinear fit, does not have this constraint. If a knowledge of mechanisms or a large data base dictates that this plot should be linear, then perhaps the flexibility of the SINH model should be suppressed to require a linear relationship. Application of these two models to other materials has to be attempted first, however, before further guidelines and constraints can be developed for their application to engine alloys.

Conclusions

The two models evaluated, MSE and SINH, have the capability to model crack growth rate behavior in elevated temperature turbine engine materials over ranges of temperature, frequency, stress ratio, and sustained load hold time. The SINH model has been formulated in a manner that provides the capability for modeling any material having the general characteristics of a number of nickel-base superalloys used in its development. The MSE model has been formulated for only one material. The methodology for its application to other materials has not been established. Both models are highly empirical in nature and do not have any inherent mechanism of material behavior built into them. However, they both have sufficient flexibility to model a large body of experimental data covering wide ranges of the four test variables. Because of the significant differences in the stages of development and experience in application of the two models, it is difficult to compare them directly. The SINH model is formulated in a computer code that has built-in functional relationships between the parameters and the test variables and that contains numerical procedures for regression analyses to determine these parameters from experimental data. The MSE model contains no such generalized relationships or procedures for determining these parameters from experimental data. Neither model has been applied extensively to materials that exhibit significant amounts of time-dependent behavior.

Acknowledgments

This work was part of a M.S. thesis by the third author at the Department of Aeronautics and Astronautics, Air Force Institute of Technology and was supported, in part, by the Metals Behavior Branch, AFWAL Materials Laboratory, Under Project 2307P1.

The authors would like to express their appreciation to Mr. James Larsen of the Materials Laboratory for this assistance in the use of the SINH model. Special thanks are extended to Dr. Jim Laflen and Mr. David Utah of General Electric Co., Evendale, Ohio, for their invaluable assistance with the MSE computer program. The support of the U.S. Air Force, under Project 2307P1, is also gratefully acknowledged.

References

[1]Larsen, J. M. and Nicholas, T., "Cumulative Damage Modeling of Fatigue Crack Growth," *Engine Cyclic Durability by Analysis and Testing*, North Atlantic Treaty Organization, AGARD CP 368, 1984, pp. 9-1—9-15.

[2]Nicholas, T., Weerasooriya, T., and Ashbaugh, N. E., "A Model for Creep/Fatigue Interactions in Alloy 718," *Fracture Mechanics: Sixteenth Symposium*, edited M. F. Kanninen and A. T. Hopper, ASTM STP 868, American Society for Testing and Materials, Philadelphia, 1985 (in press).

[3]Nicholas, T. and Weerasooriya, T., "Hold-Time Effects in Elevated Temperature Fatigue Crack Propagation," *17th National Symposium on Fracture Mechanics, 1984*, ASTM STP, American Society for Testing and Materials, Philadelphia, 1985 (in press).

[4]Stucke, M., Khobaib, M., Majumdar, B., and Nicholas, T., "Environmental Aspects in Creep Crack Growth in a Nickel Base Superalloy," *Proceedings, Sixth International Conference on Fracture*, Vol. 6, 1984 (in press).

[5]Bain, K. R. and Pelloux, R. M., "Effect of Oxygen on Creep Crack Growth in PM/HIP Nickel-base Superalloys," *Proceedings of the 5th International Symposium on Superalloys*, American Society for Metals, Metals Park, Ohio, 1984, pp. 387-396.

[6]Ashbaugh, N. E., "Creep Crack Growth Behavior in IN718 in Lab Air," *17th National Symposium on Fracture Mechanics, 1984*, ASTM STP, American Society for Testing and Materials, Philadelphia, 1985 (in press).

[7]Utah, D. A., "Crack Growth Modeling in an Advanced Powder Metallurgy Alloy," AFWAL-TR-80-4098, 1980.

[8]Larsen, J. M., Schwartz, B. J., and Annis, C. G. Jr., "Cumulative Damage Fracture Mechanics Under Engine Spectra," AFML-TR-79-4159, 1980.

[9]Annis, C. G. Jr., Wallace, R. M., and Sims, D. L., "An Interpolative Model for Elevated Temperature Fatigue Crack Propagation," AFML-TR-76-176, Pt. I, 1976.

[10]Wallace, R. M., Annis, C. G. Jr., and Sims, D. L., "Application of Fracture Mechanics at Elevated Temperature," AFML-TR-76-176, Pt. II, 1976.

[11]Sims, D. L., Annis, C. G., and Wallace, R. M., "Cumulative Damage Fracture Mechanics at Elevated Temperature," AFML-TR-76-176, Pt. III, 1976.

[12]Miller, M. S. and Gallagher, J. P., "An Analysis of Several Fatigue Crack Growth Rate Descriptions," *Fatigue Crack Growth Measurement and Data Analysis*, edited by S. J. Hudak, Jr. and R. J. Bucci, ASTM STP 738, American Society for Testing and Materials, Philadelphia, 1981, pp. 205-251.